Mechanical Properties of Fluids

Fluid Statics

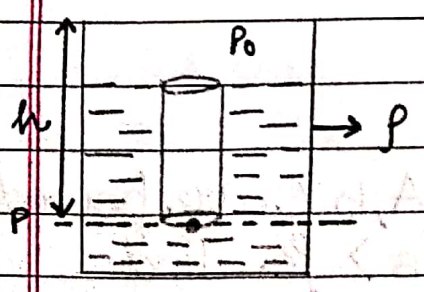
ideal fluid [liquid + gases] → (1) Incompressible [Fixed volume] [Fixed mass, constant density]
(2) Non-viscous [There is no tangential force among liquid layer] no friction

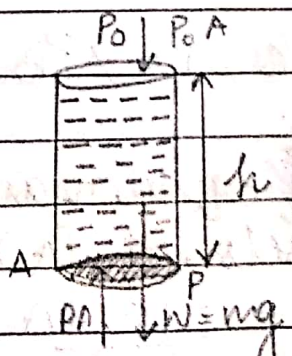
Pressure → $\frac{F_\perp}{|A|}$, $\frac{\text{Thrust}}{\text{Area}}$

→ Scalar quantity
→ Isotropic
→ SI-unit : N/m^2 or Pascal (Pa)
1 atm = 1.013×10^5 Pa, 1 atm = 760 mm of Hg
1 bar = 10^5 N/m^2, = 760 torr

→ **Volume** $1\,L = 10^{-3}\,m^3 = 10^3\,cm^3 = 1\,dm^3$

Variation of Pressure vertically with depth

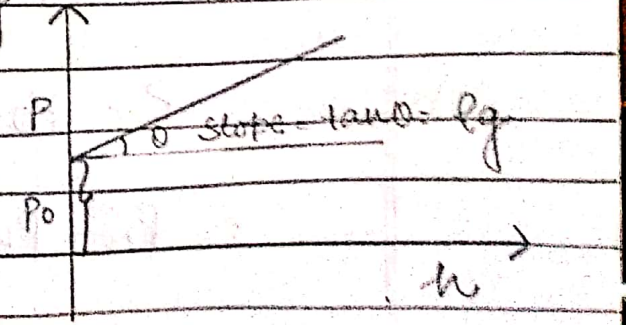




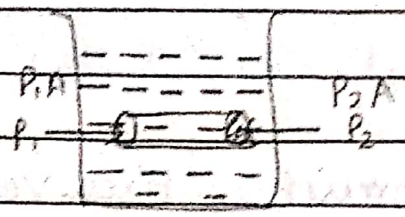
$P_0A + mg = PA$
$PA = P_0A + (\rho \times A \times h)g$
$P = P_0 + \rho gh$
Pressure at point ↓ atmospheric ↓ Pressure due to liquid column of height (h)

$m = \rho \times vol$
$m = \rho \times (A \times h)$

$\Delta P = \rho gh$



Variation of Pressure horizontally.

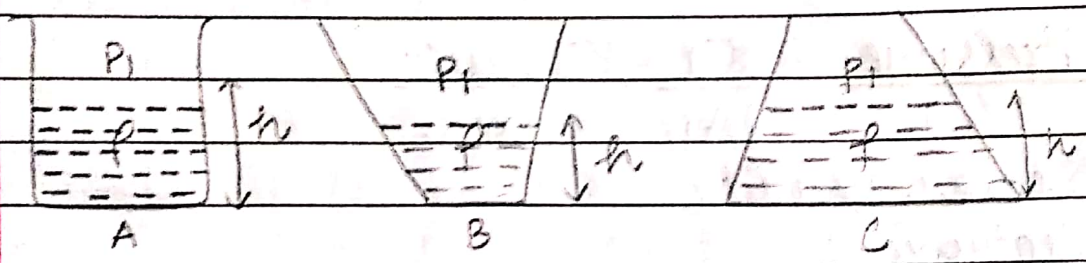


Net $\vec{a} = 0$

$P_2 A = P_1 A$

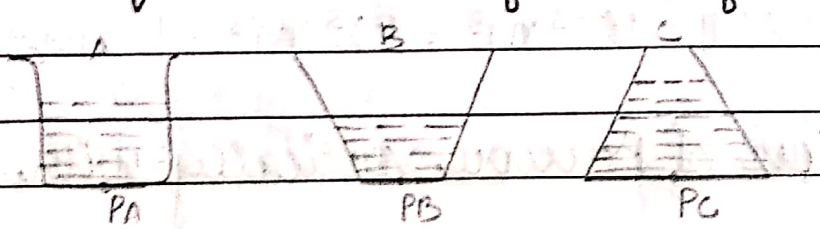
$\boxed{P_2 = P_1}$

Q.b Shape of vessel does not cause any effect on the final value of pressure.



$P_A = P_B = P_C$

Q-1 (i) Three vessels having equal base area are filled with equal volume of same fluid.



$V = A \times h$

~~$P_C > P_B > P_C$~~ $P_C > P_A > P_B$

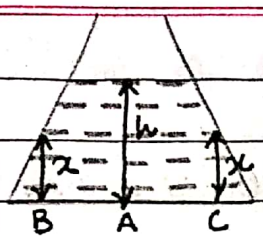
(ii) Three fluids of equal mass A, B, C are poured in 3 similar vessels. If $\rho_A > \rho_B > \rho_C$. Find.

$$P = P_0 + \frac{\rho g h \times A}{A}$$

$$P = P_0 + \frac{\rho (h \times A) g}{A} \quad = \quad P = P_0 + \frac{mg}{A}$$

$P_A = P_B = P_C$

(iii)



$P_A = P_B = P_C$

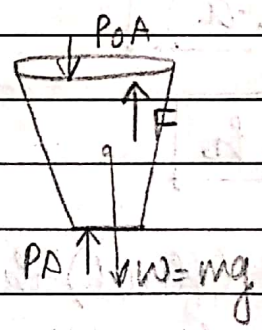
Q-2 A glass of water (1L) has height 10 cm, base area = 10 cm^2 and top area = 30 cm^2, $\rho = 10^3$ kg/m^3
1 atm = 1.01×10^5 N/m^2

(i) Find the force on bottom of glass
(ii) Find the force on water by sides of glass.

(i) $P = P_0 + \rho g h$
$= 1.01 \times 10^5 + 10^3 \times 10 \times 10$ (10 cm = 0.1 m)
$= 10^3 (1 + 1.01 \times 10^2)$
$= 10^3 (1 + 101) = 102 \times 10^3$ N/m^2

$F = PA = 102 \times 10^3 \times 10 \times 10^{-4}$
$F = 102$ N.

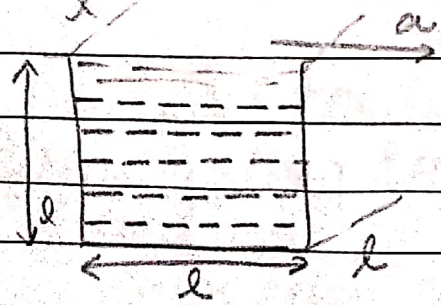
(ii)



$P_0A = 1.01 \times 10^5 \times 30 \times 10^{-4}$
$= 303$ N
$W = mg = V\rho g h$
$= 10^{-3} \times 10^3 \times 10$
$= 10$ N

$303 \text{ N} + 10 \text{ N} = 102 \text{ N} + F$
$F = 211$ N.

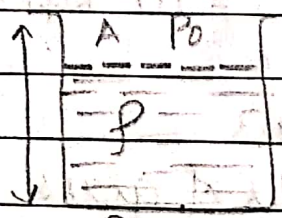
Q-3



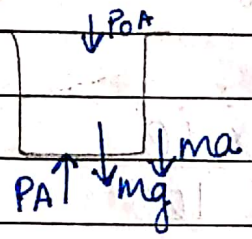
If half of total volume of water comes out what is the acceleration of vessel.

Variation in Pressure in accelerated fluids.

Case I: Vertical



$\uparrow a$ from the frame of vessel liquid $\rightarrow$ Rest



$m = \rho V$
$= \rho(A\Delta h)$

$P_0A + mg + ma = PA$

$P_0A + m(g+a) = PA$

$P_0A + \rho A\Delta h(g+a) = PA$

$P_0 + \rho h(g+a) = P$

$$\boxed{P = P_0 + \rho h(g+a)}$$

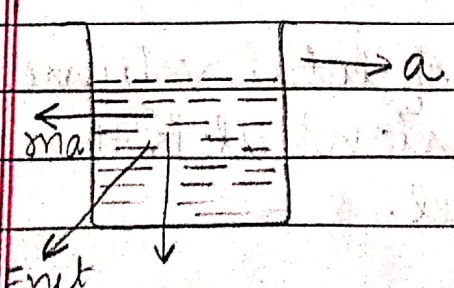
$\downarrow a$

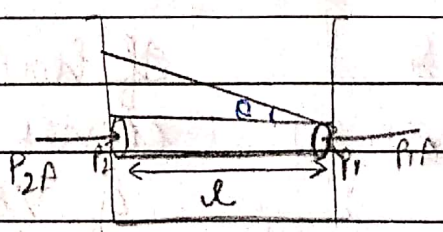
$$\boxed{P = P_0 + \rho h(g-a)}$$

$$\boxed{P = P_0 + \rho g_{eff} \cdot h}$$

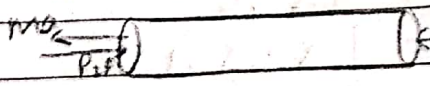
Note: In any free surface of liquid (fluid) the resultant force on surface is always $\perp$ to surface.

Case II: Horizontal





from vessel's frame,



$P_2A = P_1A + ma$

$(P_2 - P_1)A = Alρa$

$$\boxed{(P_2 - P_1) = ρla}$$

Inclination of the fluid,

$$\boxed{\tan\theta = \frac{h}{l} = \frac{a}{g}}$$

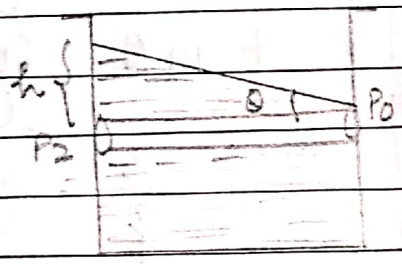
$P_2 = P_0 + ρgh$

$P_1 = P_0$

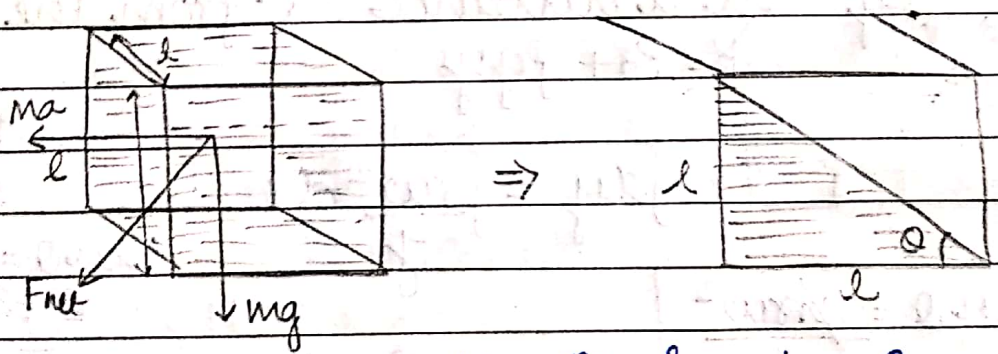
$P_2 - P_1 = lρa$

$ρgh = lρa$

$$\frac{h}{l} = \frac{a}{g}$$



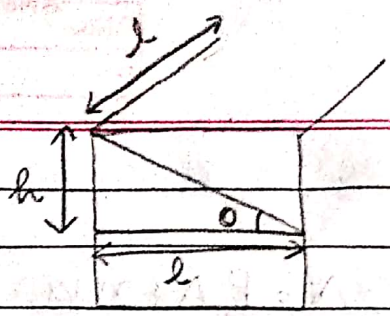
Sol-3



$$\tan\theta = \frac{l}{l} = \frac{1}{} = \frac{a}{g}$$

$$\boxed{a = g}$$

Q-4 A cubical container with side 'l' is completely filled with water. Calculate the horizontal 'a' of container so that 1/3rd of its total volume of water come out of the container.



Volume missing
$= A \times h$
$= \frac{1}{2} \times l \times h \times l$

$$V = \frac{l^2 h}{2}$$

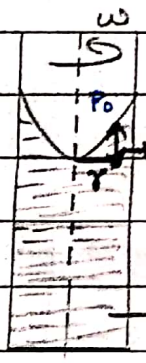
$$\frac{l^3}{3} \times 1 = \frac{l^2 h}{2}$$

$$h = \frac{2l}{3} \qquad \frac{h}{l} = \frac{2}{3}$$

$$\tan\theta = \frac{h}{l} = \frac{2}{3}$$

$$\frac{a}{g} = \frac{2}{3} \qquad a = \frac{2g}{3}$$

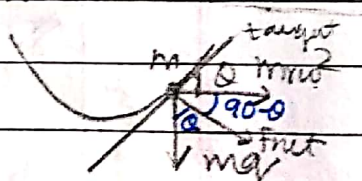
Q-5



Find the expression for pressure at a distance "r" from the axis

$$P = P_0 + \rho g y$$

$$\int dy = \int \frac{r\omega^2}{g} dr$$



$\tan\theta$ = slope $= \frac{dy}{dx}$

$$\tan\theta = \frac{r\rho r\omega^2}{\rho g}$$

$$\tan\theta = \frac{r\omega^2}{g}$$

$$\frac{dy}{dx} = \frac{r\omega^2}{g}$$

$x = r$

$$dy = \frac{r\omega^2}{g} dr$$

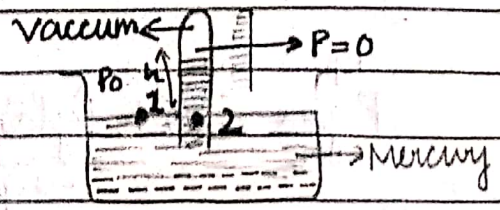
$$y = \frac{\omega^2}{g} \frac{r^2}{2}$$

$$P = P_0 + \rho g \frac{r^2\omega^2}{2g}$$

$$\boxed{P = P_0 + \frac{\rho r^2 \omega^2}{2}}$$

P.No.-
Dt.-

Barometer → Measure atmospheric Pressure
By Torricilli



$P_1 = P_2$

$P_o = \rho g h$

↳ density of mercury (13600 kg/m³) densest liquid

$P_2 = 0 + \rho g h$

At sea level,

$h = 76\ cm$

$P_o = 13600 \times 10 \times \frac{76}{100} \approx 1.01 \times 10^5\ N/m^2$ (1 atm)

[1 atm = 76 cm of Hg.]

Why Mercury is used ?

1- Densest 2- Opaque / shining 3- not vapourise

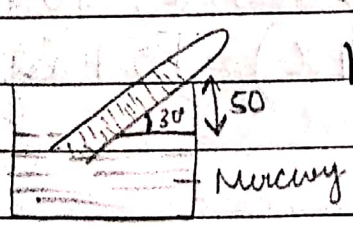
Let us suppose water is used,

$P_o = \rho_w g h_w$

$1.01 \times 10^5 = 10^3 \times 10 \times h$

$h = 10.1\ m.$

Q-6 What is the reading of barometer?



$\sin 30° = \frac{P}{h}$ $\quad \frac{1}{2} = \frac{50}{x}$

$x = 100\ cm$

$P_o = \rho g h$ (vertical height) =

$= 13600 \times 10 \times \frac{50}{100}$

$= 68000\ N/m^2$

Manometer → To measure pressure of an enclosed gas.

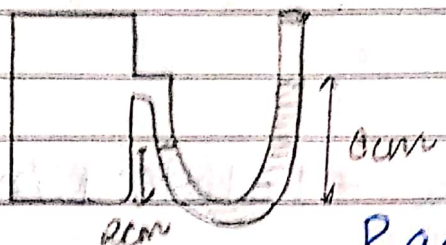
$P_1 = P_2$

$P_{gas} = P_0 + \rho g h$ → gauge Pressure

$P_2 = P_0 + \rho g h$

$P_{gas} - P_0 =$ gauge Pressure

Q-7



$P_{gas} = ?$

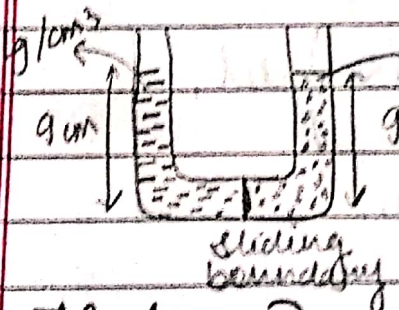
$P_{gas} = P_0 + \rho g h$

$P_{gas} = 1.01 \times 10^5 + 136 \times 10^2 \times 10 \times \frac{6}{100}$

$P_{gas} = 1.09 \times 10^5$

U-Tube Problems:

Q-8 $\rho = 1\ g/cm^3$ $\rho = 2\ g/cm^3$



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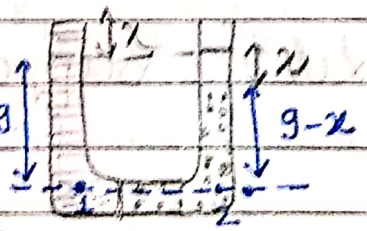

Find x ?

$P_1 = P_2$

$P_0 + 1 \times g \times (9+x) = P_0 + 2 \times g \times (9-x)$

$P_0 + g(9+x) = P_0 + 2g(9-x)$

$9g + gx = 18g - 2gx$

$gx + 2gx = 18g - 9g$

$3gx = 9g$

$\boxed{x = 3}$

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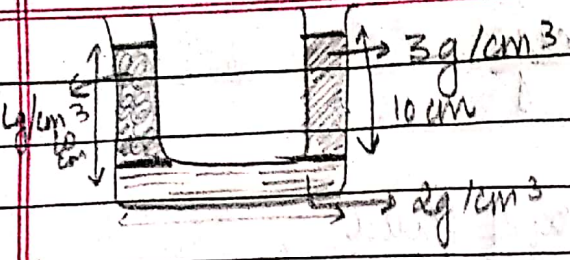
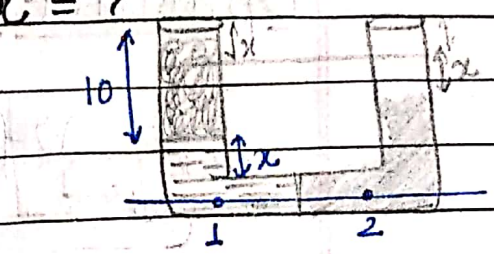
Q-9



Find $x = ?$



$P_1 = P_2$

$P_0 + 1 \times g \times 10 + 2 \times g \times x = P_0 + 3 \times g \times (10 - x)$

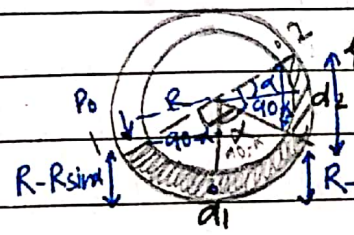
$P_0 + 10g + 2gx = P_0 + 30g - 3gx$

$2gx + 3gx = 30g - 10g$

$5gx = 20g$

$x = 4\,cm$

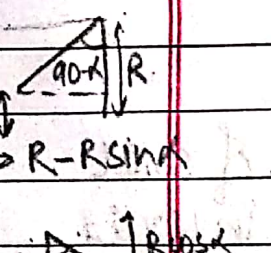
Q-10
JEE Mains 2014



Each liquid subtend 90° at the centre. Find the ratio of d_1/d_2.

$R(\sin\alpha + \cos\alpha)$

$P_0 + d_1 g R(1 - \sin\alpha) = P_0 + d_2 g R(\sin\alpha + \cos\alpha) + d_1 g R(1 - \cos\alpha)$



~~$d_1 - d_1 \sin$~~

$d_1(1 - \sin\alpha) = d_2(\sin\alpha + \cos\alpha) + d_2(1 - \cos\alpha)$

~~d_1~~ $- d_1 \sin\alpha = d_2 \sin\alpha + d_2 \cos\alpha + $ ~~d_2~~ $- d_1 \cos\alpha$

$d_1 \cos\alpha - d_1 \sin\alpha = d_2 \sin\alpha + d_2 \cos\alpha$

$d_1(\cos\alpha - \sin\alpha) = d_2(\sin\alpha + \cos\alpha)$

$\frac{d_1}{d_2} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha}$

$\frac{d_1}{d_2} = \frac{\tan\alpha + 1}{1 - \tan\alpha}$

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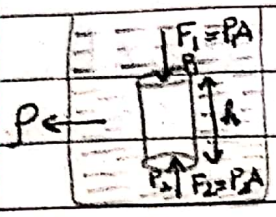
Upthrust / Buoyancy

$U = V_{\text{solid immersed}} \rho g$ ← density of fluid



$P = \frac{F_\perp}{A}$

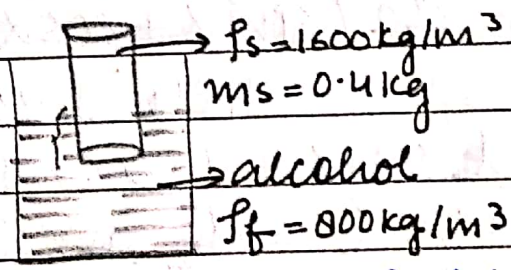
$P_2 - P_1 = \rho g h$

Net force on cylinder by fluid
$= F_2 - F_1$ (up)
$= P_2A - P_1A$
$= A(P_2 - P_1)$
$= A\rho gh$
$= (Ah)\rho g$
$= V\rho g$

Apparent weight

Wt in air − Upthrust
Loss in wt = Upthrust

Q-11



$\frac{1}{5}$th of volume of solid is immersed in liquid. Calculate apparent wt.

Ap. wt = W − V

$V_s = \frac{0.4}{16000} = 4 \times 10^{-3} m^3$

$U = V \rho_f g$

$U = \frac{1}{5}(V_s)\rho_f g$

$U = \frac{1}{5} \times \frac{4 \times 10^{-3}}{4} \times 800 \times 10$

Ap. wt = Wt in air − U
= 4 − 0.4
= 3.6 N

Archimede's Principle

$U = V \rho g$
(Volume of fluid displaced; ρ → fluid)

$m = \rho \times V$

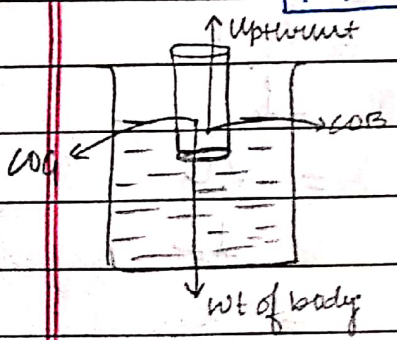
$U = m.g$
↳ mass of fluid displaced

U = weight of fluid displaced.

"When a body is partially or wholly submerged in a fluid then it experiences an upthrust which is equal to the weight of fluid displaced by submerged part of solid."

Law of flotation - Equilibrium

$F_{net} = 0$



Wt of body = upthrust (general)

Condition for flotation

$\rho_s < \rho_f$

sink → $\rho_s > \rho_f$

limiting flotation → $\rho_s = \rho_f$

Short trick

Wt. of body = upthrust

$mg = V_f \rho_f g$

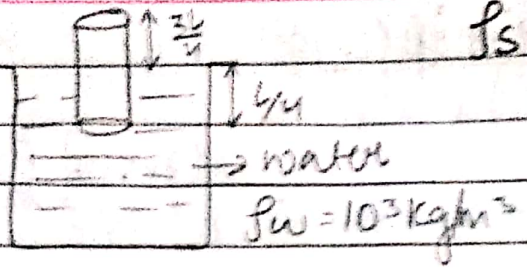
$V_s \rho_s g = V_f \rho_f g$

$V_s \rho_s = V_f \rho_f$

$A h_s \rho_s = A h_f \rho_f$

$h_s \rho_s = h_f \rho_f$

Q-12



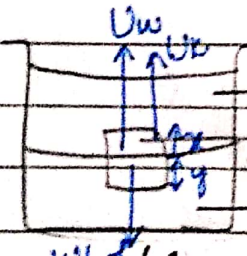
$\rho_s = ?$

$$h_s \rho_s = h_w \rho_w$$
$$L \times \rho_s = \frac{L}{4} \times 1000$$
$$\rho_s = 250 kg/m^3$$

Q-13 An iceberg ($0.9 g/cm^3$) floats on sea water ($1.1 g/cm^3$). Find the fraction of ice which is visible to a ship above sea level.

$$h_s \times \rho_s = h_w \rho_w$$
$$L \times 0.9 = h_w \times 1.1$$
$$h_w = \frac{0.9}{1.1} L = \frac{9}{11} L$$
$$L' = L - \frac{9}{11} L = \frac{11L - 9L}{11} = \frac{2L}{11}$$
$$\text{fraction} \quad \frac{2L}{11 \times L} = \frac{2}{11}$$

Q-14



What is the ratio of length of ice which is present in water to k.oil (ice is in equilibrium)

P.No. ANMOL
Dt.

$W_{ice} = V_w + V_{K\text{-}oil}$

$m_{ice} g = V_w \rho_w g + V_{K\text{-}O} \times \rho_{K\text{-}O} \times g$

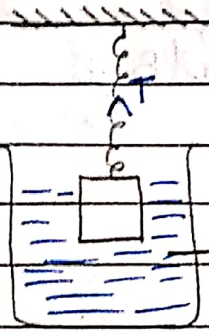
$V_{ice}\, \rho_{ice} = V_w \rho_w + V_K \rho_K$

$0.9 \times A(x+y) = Ay\ 10 + Ax\ 0.8$

$9x + 9y = 10y + 8x$

$\boxed{x = y}$

Q-15



$r = 5cm$ (Equilibrium)

$h = 20cm$ Find the elongation in string.

$\rho_s = 8000\ kg/m^3$

$\rho_w = 10^3\ kg/m^3$

$K = 500\ N/m$

Wt. of body = Upthrust + T

$V_s \times \rho_s \times g = V_f \times \rho_f \times g + T$

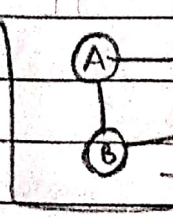
$1.5 \times 10^{-3} \times 8000 \times 10 = 1.6 \times 10^{-3} \times 1000 \times 10 + T$

$T = 8\ N$

$Kx = 8$ $\quad T = 112\ N$

$x = \frac{8}{500}$ $\quad x = \frac{112}{500} = 23cm$

Q-16
JEE [2011]

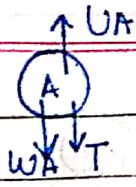


The system is in equilibrium and string has tension then which is/are correct.

(a) $d_A < d_f$ (b) $d_A > d_f$

(c) $d_B > d_f$ (d) $d_A + d_B = 2d_f$

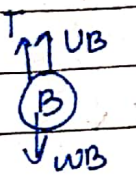
Dt.-



$U = W_A + T$

$V\rho g = V\rho_A g + T$

$V_{dF} g = V_{dA} g + T \quad —(i)$



$T + U_B = W_B$

$T + V_{dF} g = V_{dB} g \quad —(ii)$

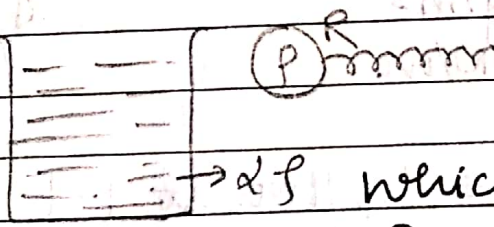
$V_{dF} g = V_{dA} g + T$

$+ V_{dF} g = V_{dB} g - T$

$2V_{dF} g = V_{dA} g + V_{dB} g$

$$\boxed{d_A + d_B = 2d_F}$$

Q-17
IIT JEE 2013 Adv.



which of the following is are correct?

(a) elongation of spring $\frac{4\pi R^3 \rho g}{3K}$

(b) elongation of spring $\frac{8\pi R^3 \rho g}{3K}$

(c) lighter sphere is partially submerged

(d) lighter sphere is completely submerged.

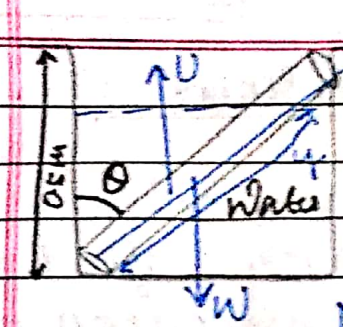
$V 2\rho g - V\rho g = T$

$V\rho g = T$

$T = \frac{4}{3}\pi R^3 \rho g$

$\& \chi = \frac{4\pi R^3 \rho g}{3K}$

Q-18



Find θ

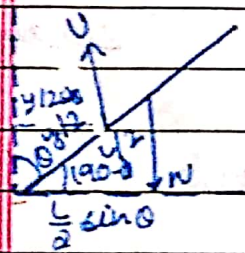
$N\cos\theta + U = W$

$N\left(\frac{0.5}{y}\right) + V_f \rho_f g = V_s \rho_s g$

$N\left(\frac{0.5}{y}\right) + Ay\rho_f g = A(1)\times 0.5\times10^3\times g$

Rotational equilibrium

$\tau_{net} = 0 \quad \tau_p = 0$



$\tau_W = \tau_U$

$W\frac{L}{2}\sin\theta = U\frac{y}{2}\sin\theta$

$A\times0.5\times10^3\times g\times L = Ay\,10^3 g\,y$

$0.5\times(1)^2 = y^2$

$y = \sqrt{0.5}$

$\cos\theta = \frac{0.5}{\sqrt{0.5}} = \sqrt{0.5} = \sqrt{\frac{1}{2}}$

$\cos\theta = \frac{1}{\sqrt{2}} \quad \theta = 45°$

Fluid Dynamics

Ideal fluid flow.

→ Incompressible (No change in density)
→ Non-viscous (No friction b/w layers)
→ Irrotational $\omega \to 0$

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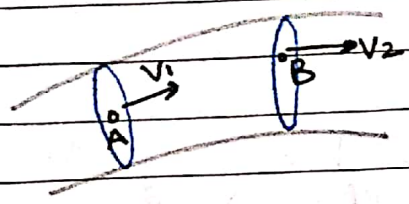
Steady flow / Streamline flow



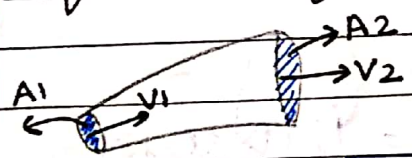
(i) At any point in space the velocity of fluid molecules remains constant with time.

(ii) Any particle follows the path of previous particle.

(iii) Two streamlines never intersect.

Equation of continuity

[Ideal fluid / streamline flow]



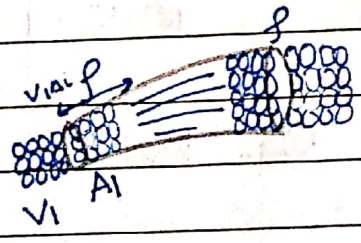
$A_1V_1 = A_2V_2$

AV = constant

$$v \propto \frac{1}{\text{Area}}$$

Rate of flow = volume flown in 1 sec.

(AV)



In time Δt, volume of fluid that enters the

$d = V_1 \Delta t$

Volume $= A_1V_1\Delta t$

Mass of volume that entirs in Δt $= \rho \times$ volume

$= \rho A_1V_1\Delta t$

In time Δt, vol of fluid that leaves $= A_2V_2\Delta t$

Mass $= \rho A_2V_2\Delta t$

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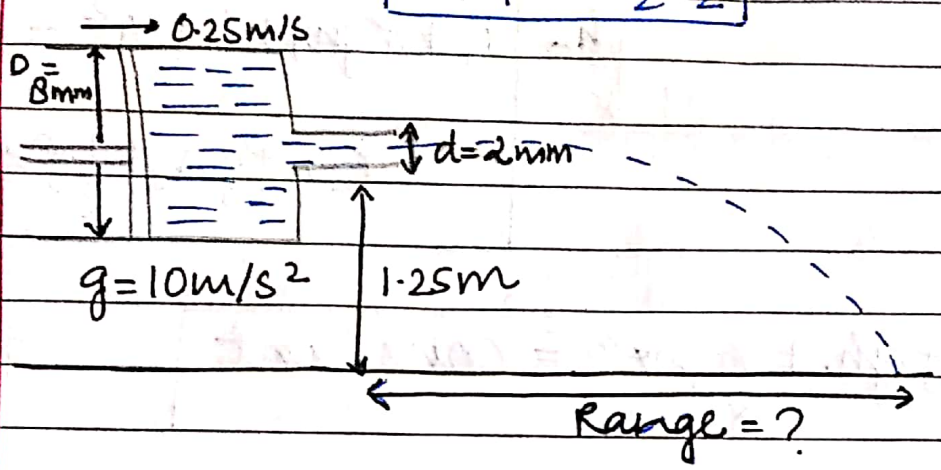
Dt.-

Conservation of Mass.

$$\rho A_1 V_1 \Delta t = \rho A_2 V_2 \Delta t$$

$$\boxed{A_1 V_1 = A_2 V_2}$$

Q-19
IIT JEE
2004



$$A_1 V_1 = A_2 V_2$$

$$\pi \times (0.08)^2 \times 0.25 = \pi \times (0.02)^2 \times V_2$$

$$V_2 = 4 m/s$$

$$S_y = u_y t - \frac{1}{2} g t^2$$

$$-1.25 = 0 - \frac{1}{2} \times 10 \times t^2$$

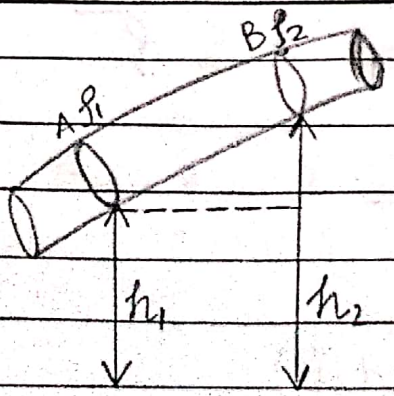
$$t = 0.5 \text{ seconds}.$$

$$S_x = u_x t + \frac{1}{2} a t^2$$

$$S_x = 4 \times 0.5^2 + 0$$

$$S_x = 2 m.$$

Bernoullis Theorem

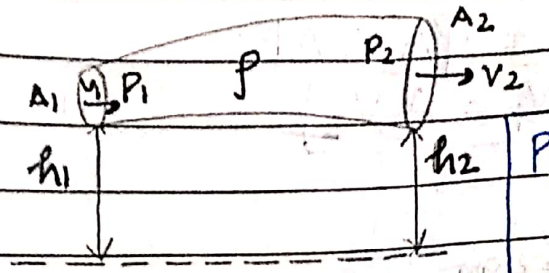


$$P_2 - P_1 = \rho g (h_2 - h_1)$$

static fluid.

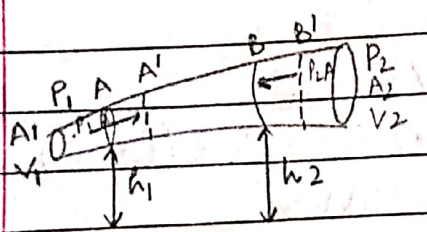
$$\Delta P = \rho g h$$

Ideal fluid flow / streamline



$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$



We will consider fluid contained in between AB.

In time interval Δt, this fluid section moves to A'B'

Work Energy Theorem,

$W_{By\ all\ forces} = \Delta KE$

Work done by fluid Pressure

$$W_f = W_{AA'} + W_{BB'}$$
$$= P_1 A_1 v_1 \Delta t - P_2 A_2 v_2 \Delta t$$
$$= P_1 \frac{\Delta m}{\rho} - P_2 \frac{\Delta m}{\rho} = \frac{\Delta m}{\rho}(P_2 - P_1)$$

Work done by gravity

$$W_c = -\Delta U$$
$$W_{gravity} = -(U_f - U_i)$$

$= U_{AB} - U_{A'B'}$

$= U_{AA'} + U_{A'B} - U_{A'B} - U_{BB'}$

$= U_{AA'} - U_{BB'}$

$= \Delta m g h_1 - \Delta m g h_2$

$= \Delta m g (h_1 - h_2)$

Work done by all forces = ΔKE

$W_f + W_g = K_f - K_i$

$= K_{A'B'} - K_{AB}$

— do — $= K_{A'B} + K_{BB'} - K_{AA'} - K_{A'B}$

$= K_{BB'} - K_{AA'}$

$W_f + W_g = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2$

$\Delta m (P_1 - P_2) + \Delta m g (h_1 - h_2) = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2$

$P_1 - P_2 + \rho g h_1 - \rho g h_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$

$$\boxed{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + P_2}$$

→ Pressure head + Potential head + Kinetic head = constant

→ $\frac{P}{\rho g} + h + \frac{v^2}{2g}$ = constant

→ Bernoulli

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Dt.-

Law : Conservation of energy

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

flowing fluid → 3 types of energy

① Pressure energy → W Pressure force = PAx

$$\frac{\text{Pressure energy}}{\text{volume}} = \frac{PAx}{Ax} = P$$

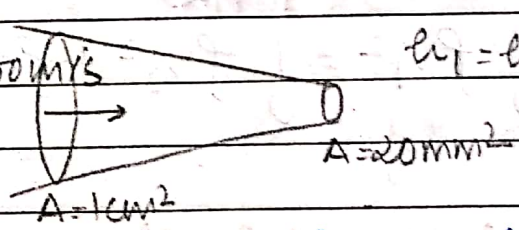
② Potential energy → mgh

$$\frac{\text{Potential energy}}{\text{volume}} = \frac{mgh}{Vol.} = \rho gh$$

③ Kinetic energy → $\frac{1}{2}mv^2$

$$\frac{K.E}{\text{volume}} = \frac{\frac{1}{2}mv^2}{Vol} = \frac{1}{2}\rho v^2$$

Q-20



$h_1 = h_2$ Find the pressure diff. b/w 1 and 2 if the fluid has $\rho = 1200\ kg/m^3$

$$A_1 v_1 = A_2 v_2$$

$$1\times10^{-4}\times100 = 2\times10^{-5}\times v_2$$

$$v_2 = 500\ m/s$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

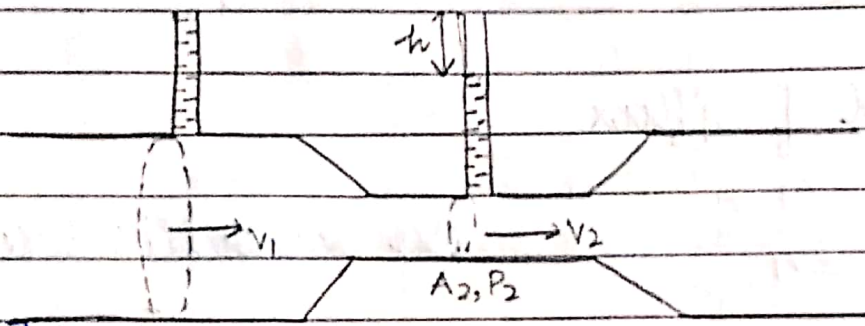
$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$P_1 - P_2 = 14.4\times10^7\ Pa$$

Application of Bernoulli's theorem

I. Venturimeter

Measure rate of flow of liquid through a tube.. [Speed of flowing liquid]



$[h_1 = h_2]$

By Bernoulli's theorem,

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$[P_1 - P_2 = \rho g h]$

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$\rho g h = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$v_2^2 - v_1^2 = 2gh \quad \text{---(i)}$$

By equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$$\boxed{\frac{v_2}{v_1} = \frac{A_1}{A_2}} \quad \text{---(ii)}$$

From eq^n (i) $\rightarrow$ $v_1^2 \left(\left(\frac{v_2}{v_1} \right)^2 - 1 \right) = 2gh$

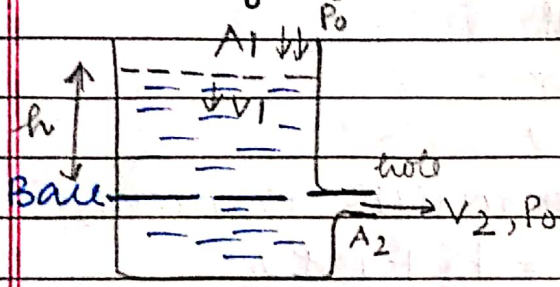
$$v_1^2 \left(\left(\frac{A_1}{A_2} \right)^2 - 1 \right) = 2gh$$

$$v_1^2 = \frac{2gh \times A_2^2}{A_1^2 - A_2^2}$$

$$V_1 = \left(\sqrt{\frac{2gh}{A_1^2 - A_2^2}}\right) A_2$$

$$V_2 = \left(\sqrt{\frac{2gh}{A_1^2 - A_2^2}}\right) A_1$$

II. Speed of efflux



By Bernoulli's theorem,

$$P_0 + \rho gh + \frac{1}{2}\rho V_1^2 = P_0 + \rho g(0) + \frac{1}{2}\rho V_2^2$$

$$\rho gh + \frac{1}{2}\rho V_1^2 = \frac{1}{2}\rho V_2^2$$

Assumption,

$$A_1 V_1 = A_2 V_2$$

$$\frac{V_1}{V_2} = \frac{A_2}{A_1}$$

$\{A_2 <<< A_1\}$

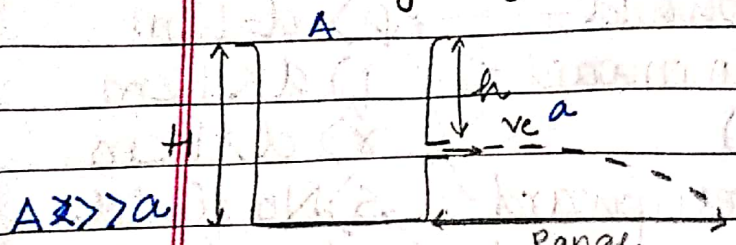
$V_1 \to 0$

$$\rho gh = \frac{1}{2}\rho V_1^2$$

$$V_{0,e} = \sqrt{2gh}$$

(Torricelli's theorem)

Range of fluid



$$-S_y = 0 - \frac{1}{2}gt^2$$

$$H-h = \frac{1}{2} \times gt^2$$

$$t = \sqrt{\frac{2(H-h)}{g}}$$

$$S_x = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$$

$$(S_x)^2 = 2gh \times \frac{2H-2h}{g}$$

$$S_x^2 = 4Hh - 4h^2$$

$$S_x = \sqrt{4h(H-h)}$$

$$\text{Range} = \sqrt{4h(H-h)}$$

→ Range depends upon position of orifice.

Maximum Range,

$R^2 \rightarrow$ Max

$$\frac{dR^2}{dh} = 0$$

$$R^2 = 4h(H-h)$$
$$= 4hH - 4h^2$$
$$= 4H - 8h$$
$$4H = 8h$$

$$h = \frac{H}{2}$$

$$R_{max} = \sqrt{4 \times \frac{H}{2} \times \frac{H}{2}} = R_{max} = H$$

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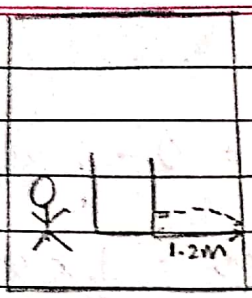
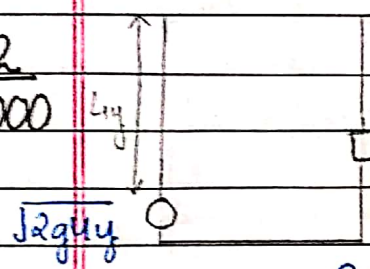
Q-21
JEE Adv. 2014



List I
A) Lift a upward
B) Lift a downward (a<g)
C) Lift moves upward with constant speed
d) Free fall of lift

List II
P) d=1.2m
q) d<1.2m
r) d>1.2m
s) No fluid falls out.

Q-22
JEE 2000



if the rate of fluid coming out from both orifice is same then find R in terms of L

Rate of flow$_1$ = Rate of flow$_2$

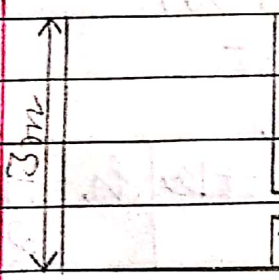
$A_1 v_1 = A_2 v_2$

$L^2 \times \sqrt{2gy} = \pi R^2 \times \sqrt{2g(L-y)} \times 2$

$R^2 = \frac{L^2}{\pi 2}$

$R = \frac{L}{\sqrt{2\pi}}$

Q-23
IIT JEE 2005



The ratio of cross sectional area of orifice to that of tank is 0.1
Find v^2

$h = 300cm - 52.5cm = 247.5cm$
$= 2.475m$

$A_1 v_1 = A_2 v_2$

$0.1 = \frac{A_2}{A_1} = \frac{v_1}{v_2}$

$v_1 = \frac{v_2}{10}$

$$P_0 + \rho g h + \frac{1}{2}\rho v_1^2 = P_0 + \rho g(0) + \frac{1}{2}\rho v_2^2$$

$$2gh + v_1^2 = v_2^2$$

$$v_2^2 - v_1^2 = 2gh$$

$$v_2^2 - \frac{v_2^2}{100} = 2gh$$

$$\frac{100 v_2^2 - v_2^2}{100} = 2gh$$

$$v_2^2 \times \frac{99}{100} = 2 \times 10 \times 2.475$$

$$v_2^2 = 50 \frac{m^2}{s^2}$$

Time taken to empty the tank

$$t = \frac{A_1}{A_2}\left(\sqrt{\frac{2h}{g}}\right)$$

Fluid Friction - Viscosity

"Friction among horizontal fluid layers called viscosity"

velocity gradient

$$\frac{\text{change in velocity}}{\text{height}} = \frac{\Delta v}{\Delta z} \rightarrow \frac{dv}{dz}$$

Viscous Force

→ Internal friction in horizontal fluid layers.

→ $f_v \propto \frac{dv}{dz}$

→ $f_v \propto A$

→ $\boxed{f_v = -\eta A \frac{dv}{dz}}$

η → coefficient of viscosity
- → depends upon liquid
- → depends upon temp ↑ η ↓

η

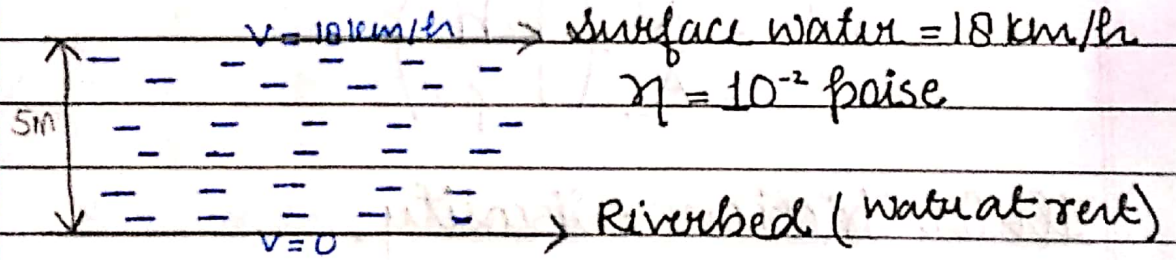
S.I unit → N-s/m²

CGS unit → dyne-s/cm² [poise]

1 poise = 10^{-1} N-s/m²

Dimension → $ML^{-1}T^{-1}$

Q-24



Find the tangential / shearing stress among layers.

$$\frac{\Delta v}{\Delta z} = \frac{\frac{18 \times 5}{18} - 0}{5} = \frac{5\,m/s}{5} = 1$$

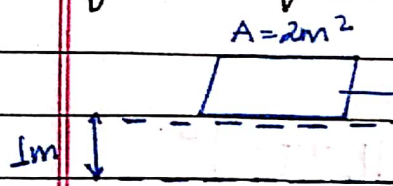
$$f_v = -\eta A \frac{dv}{dz} = 10^{-3} \times A \times 1$$

Stress = 10^{-3} N/m²

Friction between solid surface & liquid surface is contact

Case 1: Surface Contact

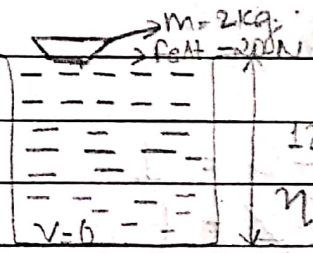
Q-25 Wooden plate $A = 2m^2$, $v = 2m/s$ fluid. $\eta = 10^{-2}$ poise. fluid depth 1m (River bed) find the force required to keep the plate in uniform motion.



$$f = \eta A \frac{dv}{dz} = 10^{-3} \times 2 \times 2 = 4 \times 10^{-3} N$$

$$F_{ext} = 4 \times 10^{-3} N$$

Q-26 Find the acceleration of boat.



$$f = \frac{10 \times 10^{-1}}{} \quad f_v = \frac{1 \times 1 \times 10}{1} = 10 N$$

$$F = 20 - 10 = 10 N$$

$$F = ma$$

$$a = 10/2 = 5 m/s^2$$

Stoke's theorem

Case 2: Solid immersed and moving in the fluid.

The viscous force on a moving object inside a fluid depends upon :-

(i) Shape & size of object.
(ii) Speed of object
(iii) Velocity of fluid (η)

Spherical object inside fluid :

Viscous force depends upon - (f_v)

(i) radius of sphere (r)
(ii) speed of sphere (v)
(iii) coefficience of viscosity (η)

$$F \propto r^a v^b \eta^c$$

$$F = k r^a v^b \eta^c$$

$$[MLT^{-2}] = [L]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c$$

$$[MLT^{-2}] = M^c L^{a+b-c} T^{-b-c}$$

$c = 1$

$-b - c = -2$

$-b - 1 = -2$

$-b = -1$

$b = 1$

$a + b - c = 1$

$a + 1 - 1 = 1$

$a = 1$

$$F_v = k r v \eta$$

$$\boxed{F_v = 6\pi \eta r v}$$

Q-27 $v = 10^3$ m/s, $r = 0.5$ m

~~$m = 6.28$ kg~~

~~$\eta = 10^{-3}$ Ns/m²~~

Find the acceleration of the object?

$$F = 0.5 \times 10^3 \times 10^{-3} \times 6\pi$$

$$F = 0.5\,N \times 6\pi = 9.42\,N$$

~~$F = ma$~~ $F = ma$

~~$a = \frac{0.5}{6.28}$~~ $a = \frac{0.5}{6.28} \frac{9.42}{} = 1.5\,m/s^2$

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sphere.

Q-28 A solid immersed in a liquid of coefficient of viscosity η, sphere of radius r mass m and moving with velocity v_0. Find the distance travel by the object before stopping.

$$a = \frac{F}{m} = \frac{-6\pi\eta r v}{m}$$

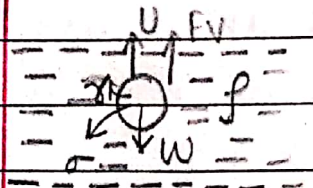
$$a = v\frac{dv}{ds} \Rightarrow \frac{-6\pi\eta r v}{m} = v\frac{dv}{ds}$$

$$-K = \frac{dv}{ds} \Rightarrow \int_{v_0}^{0} dv = -\int_0^s K\,ds = -K\int_0^s ds$$

$$0 - v_0 = -K(s-0) \Rightarrow \boxed{s = \frac{v_0}{K}}$$

$$\boxed{s = \frac{v_0 m}{6\pi\eta r}}$$

Terminal velocity (v_t)



$$W = U + Fv$$
$$mg = vfg + 6\pi\eta r v_T$$
$$\sigma vg = vfg + 6\pi\eta r v_T$$
$$v_T = \frac{v\sigma g - vfg}{6\pi\eta r}$$

$V = \frac{4}{3}\pi r^3$

$$v_T = \frac{vg(\sigma - f)}{6\pi\eta r}$$

$$v_T = \frac{\frac{4}{3}\pi r^3 g(\sigma - f)}{6\pi\eta r} = \boxed{\frac{2r^2(\sigma - f)g}{9\eta}}$$

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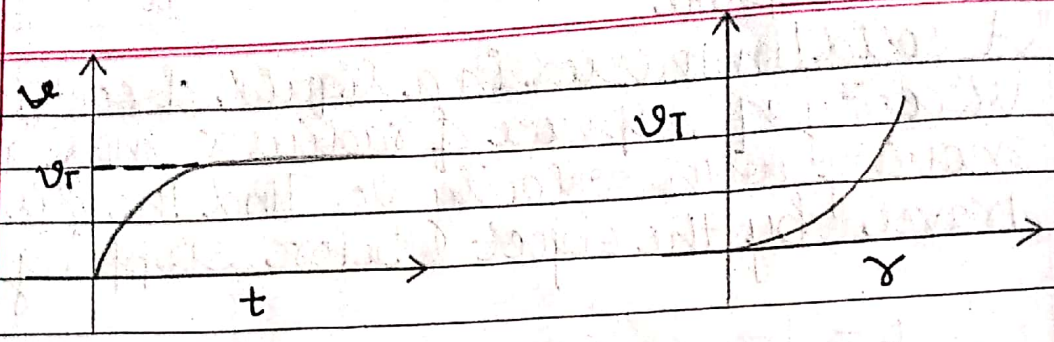
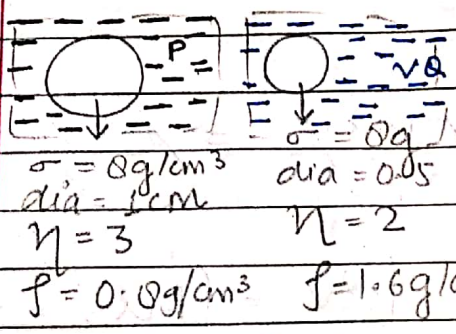


Q-29
2016 IIT JEE Adv.



Find the ratio of v_P to v_Q.

$$v_P = \frac{2}{9} \frac{0.5 \times 0.5 (8 - 0.8) g}{\eta \times 3}$$

$$v_Q = \frac{2}{9} \frac{0.25 \times 0.25 (8 - 1.6) g}{\eta \times 2}$$

$$\frac{v_P}{v_Q} = \frac{2 \times 0.5 \times 0.5 \times 7.2 g}{9 \times 3} \times \frac{9 \times 2}{2 \times 0.25 \times 0.25 \times 6.4 g}$$

$$\frac{v_P}{v_Q} = \frac{0.6}{0.2} = 3$$

Q-30 Estimate the terminal speed of a raindrop?

$\sigma = 10^3 kg/m^3$, $\rho = 1.2 kg/m^3$

$r_{drop} = 0.02 cm$, $\eta = 1.8 \times 10^{-5}$

$$v_T = \frac{2}{9} \times \frac{0.02 \times 0.02}{100 \times 100} \times \frac{(10^3 - 1.2) \times 10}{1.8 \times 10^{-5}}$$

$$v_T = 5 cm/s$$

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* In formula of v_T if, $\sigma < \rho$
v_T = -ve ex.[air bubble in water]
'-' sign signifies rises upwards.

Q-31 air bubble, $r = 0.4$ mm
$d_{air} = 1.2$ kg/m^3, $d_{fluid} = 0.9 \times 10^3$ kg/m^3
$\eta = 0.15$ N-s/m^2

$$v_T = -\frac{2}{9} \times \frac{(0.9 \times 10^3 - 1.2)}{0.15 \times 1000 \times 1000} \times 10 \times 0.4 \times 0.4$$

$$v_T = -2.13 \times 10^{-3} \text{ m/s} \simeq -20 \text{ cm/s}$$

Surface Tension

"The property of liquid surface to remain in minimum surface Area."

→ Its a property unique to fluid.

$S = F_\perp / L$
$F_\perp = S \times l$

Surface tensions of some substance :-
- water → 0.075 N/m
- Soap solution → 0.03 N/m
- Mercury → 0.4 N/m

Q-32 ~~Who~~ Water in a beaker of $r = 5$ cm. Find the force exerted by one side of water of a diameter on surface of the other side of water of diameter.
($S = 0.075$ N/m)

$$F = S \times l = \frac{0.075 \times 10}{1000} = \frac{750}{100000} = 0.0075 \text{ N}.$$

→ soap film has two layers..

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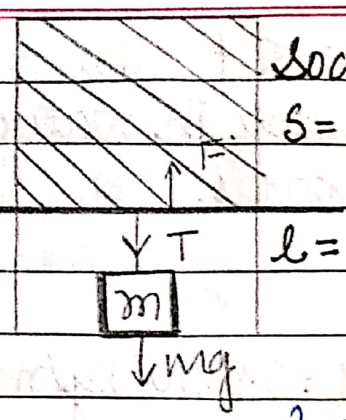
Q.33



Soap film
$S = 0.03 N/m$
$l = 10 cm.$

Find the value of 'm' such that sliding wire remains in equilibrium

$F = T$

$2 \times S \times l = m \times g$

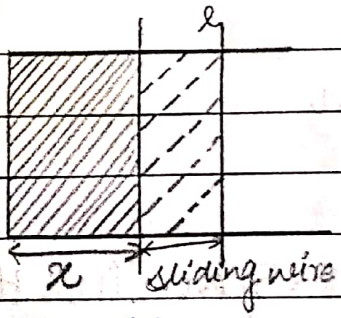
$2 \times 0.03 \times \frac{10}{100} = m \times 10$

$m = 0.0003 kg.$

Surface Energy U, ΔU

→ "Surface Molecules have more energy than molecules in Bulk."

→ Liquid surface → More Energy, for stability it want least energy therefore liquid surface tries to minimise its surface area.



slowly
no friction

$A_i = 2lx$

$A_f = 2l(x+\Delta x)$

$\Delta A = 2l\Delta x$

Work done by external agent
$= 2F\Delta x$
$= 2S \times l \times \Delta x$
$= S(2l.\Delta x)$
$= S\Delta A$

$\boxed{\Delta U = S\Delta A}$

$\boxed{S = \frac{\Delta U}{\Delta A}} \quad \frac{J}{m^2}$

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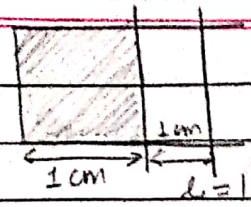
Q-34



$S = 0.04\ N/m$

Find the (i) $W = \Delta U$ [If without friction]
(ii) ΔU

$\Delta U = S \Delta A$
$= 0.04 \times 2 \times 10 \times 1 \times 10^{-4}$
$= 8 \times 10^{-5}\ J$

Q-35 Find the work req. to increase the radius of soap bubble from 2cm to 5 cm. ($S = 0.03 N/m$)

$\Delta U = S \times \Delta A$
$= 0.03 \times (4\pi R^2 - 4\pi r^2) \times 2$
$= 0.03 \times 8\pi (5^2 - 2^2) \times 10^{-4}$
$= 1.6 \times 10^{-3}\ J$

Q-36 Liquid drop of $R = 10^{-6} m$ divided into 1000 identical droplets. $S = 0.07\ N/m$.
What is the change in surface energy.

Surface Area ↑

$\Delta U = S \times \Delta A \Rightarrow 0.07 \times (1000 \times 4\pi r^2 - 4\pi R^2)$

$\Delta U = \_\_\_ = 8 \times 10^{-12}\ J$

V big drop = 1000 × V one small

$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$

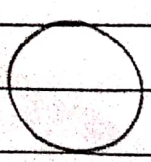
$(10^{-6})^3 = 1000 \times r^3$

$r^3 = 1 \times 10^{-18}$

$r^3 = (1 \times 10^{-6})^3$

$r = 1 \times 10^{-6}\ m.$

Q-37 IIT JEE 2017 Adv.



$R = 10^{-2} m$ → 00000 identical drops.

$K = 10\alpha$

$\alpha = ?$

$\Delta U = 10^{-3}\ J$

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37. $\frac{4}{3}\pi R^3 = 10^{\alpha} \times \frac{4}{3}\pi\gamma^3$

$10^{-6} = 10^{\alpha} \times \gamma^3$

$\gamma^3 = \frac{10^{-6}}{10^{\alpha}}$

$\gamma^{3/3} = \frac{10^{-6/3}}{10^{\alpha/3}}$

$\gamma = \frac{10^{-2}}{10^{\alpha/3}}$

$\Delta U = S \times \Delta A$

$= \frac{0.1}{4\pi} \times 4\pi \times (10^{\alpha}\gamma^2 - R^2)$

$10^{-3} = 0.1 \times \left(\frac{10^{\alpha} \times 10^{-4}}{10^{\alpha 2/3}} - 10^{-4}\right)$

~~$0.0101 = \frac{10^{\alpha-4}}{10^{\alpha 2/3}}$~~

~~$10^{\alpha 2/3} \times 0.0101 = 10^{\alpha-4}$~~

$10^{-3} = 10^{-1} R^2 \left(10^{\alpha - \frac{2\alpha}{3}} - 1\right)$

$10^{-3} = 10^{-1} \times 10^{-4} \left(10^{\frac{\alpha}{3}} - 1\right)$

$10^{-3} = 10^{-5} \left(10^{\frac{\alpha}{3}} - 1\right)$

$10^{2} = 10^{\alpha/3} - 1$

$100 = 10^{\alpha/3}$

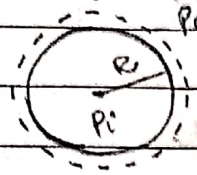
$10^2 = 10^{\alpha/3}$

$\alpha/3 = 2$

$\boxed{\alpha = 6}$

Excess Pressure In a Liquid Drop

(i) Surface Energy Method



This drop expands from R → R+ΔR

Work done in expansion = F x disp
= (Fi - Fo) x ΔR
= (Pi x 4πR² - Po 4πR²) x ΔR
= (Pi - Po) x 4πR² x ΔR

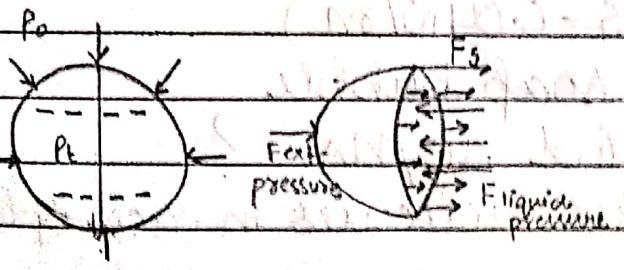
ΔU = S x ΔA
= S x (Af - Ai)
= S x (4π(R+ΔR)² - 4πR²)
= S x 4π (R² + ΔR² + 2RΔR - R²)
= S x 4π x 2RΔR

S x 4π x 2RΔR = (Pi - Po) x 4πR² x ΔR

$$(P_i - P_o) = \frac{2S}{R}$$

$$\Delta P = 2S/R$$

(ii)



(F = P x A)

Equilibrium
Fe + Fs = FL
Po x πR² + S x 2π R = Pi x πR²
2S = PiR - PoR

$$P_i - P_o = \frac{2S}{R}$$

$$\Delta P = 2S/R$$

Soap Bubble

$$(P_i - P_o) = \frac{4S}{R} \Rightarrow \Delta P = \frac{4S}{R}$$

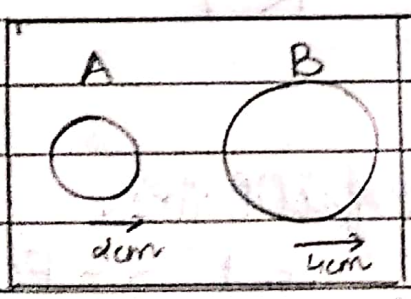
Q-38 Find the excess pressure in a
(i) ~~Liquid~~ Mercury drop ($r = 2mm$), $S = 0.465$
(ii) Soap bubble (filled with air) ($r = 4mm$) $S = 0.03$
(iii) air bubble in water ($r = 4mm$) $S_W = 0.076$

(i) $P_i - P_o = \frac{2 \times 0.465 \times 10^3}{2} = 465\ Pa$

(ii) $P_i - P_o = \frac{4 \times 0.03 \times 10^3}{4} = 30\ Pa$

(iii) $P_i - P_o = \frac{2 \times 0.076 \times 10^3}{4 \times 2} = 38\ Pa$

Q-39 IIT JEE 2009



Air chamber Pair $= 8\ N/m^2$
$(S = 0.04\ N/m)$
soap bubble
find $n_B/n_A = ?$
no. of moles of air molecules in each sphere?

$$PV = nRT$$
$$n = \frac{PV}{RT}$$
$$\frac{n_B}{n_A} = \frac{P_B V_B}{RT} \times \frac{RT}{P_A \times V_A} = \frac{P_B V_B}{P_A V_B}$$

$$P_B V_B = P_0 + \frac{4S}{R} \times \frac{4}{3}\pi (R)^3$$

$$\Rightarrow 0 + \frac{4 \times 0.04}{4 \times 10^{-2}} \times \frac{4}{3}\pi \left(\frac{4}{100}\right)^3$$

$$\Rightarrow 0 + 4 \times \frac{4}{3}\pi \left(\frac{4}{100}\right)^3 = 12 \times \frac{4}{3}\pi \left(\frac{4}{100}\right)^3$$

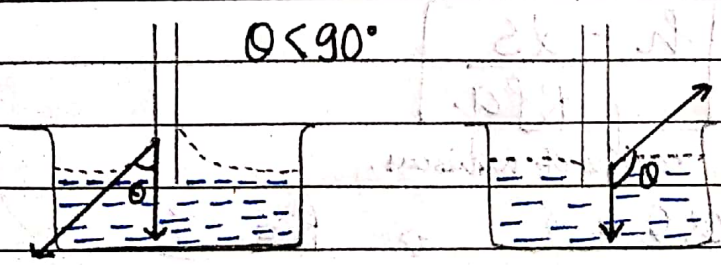
$$P_A V_B = 0 + \frac{2 \times 0.04}{2 \times 10^{-2}} \times \frac{4}{3}\pi \left(\frac{2}{100}\right)^3$$

$$= 16 \times \frac{4}{3}\pi \left(\frac{2}{100}\right)^3$$

$$\frac{n_B}{n_A} = \frac{12}{6} \times \frac{4}{3} \times \left(\frac{4}{100}\right)^3 \times \frac{3}{4} \times \left(\frac{100}{2}\right)^3 \times \frac{16}{2}$$

$$\boxed{\frac{n_B}{n_A} = 6}$$

Contact Angle 'θ' → [Tangent to liquid surface & tangent to solid surface away]



Concave miniscus
Methyl Iodide, (29°)
Impure water
→ wet the solid
→ liquid rise

Convex miniscus
Mercury → (137°)
→ Do not wet solid

[Pure water = 90°]

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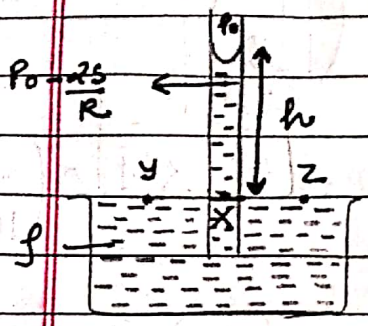
Capillary Action

The height about which substance or rise or depress is given as.



$$h = \frac{2S\cos\theta}{r\rho g}$$

S → Surface tension, r → radius of tube, ρ → density of liquid

$\theta < 90^\circ \rightarrow h = +ve$

$\theta > 90^\circ \rightarrow h = -ve$

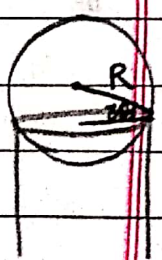
Derivation

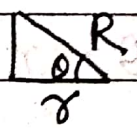
$$P_x = P_0 - \frac{2S}{R} + \rho g h$$

$$P_z = P_y = P_0$$

$$P_0 = P_0 - \frac{2S}{R} + \rho g h$$

$$\frac{2S}{R} = \rho g h$$





$$\boxed{h = \frac{2S}{R\rho g}}$$

R → radius of miniscus.

$$\cos\theta = \frac{r}{R} \Rightarrow R = \frac{r}{\cos\theta}$$

$$\boxed{h = \frac{2S\cos\theta}{r\rho g}}$$

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Q-40 Pure water $\theta = 0°$, $h = 7.5\,cm$, $S = 0.07\,N/m$, $\rho = 10^3\,kg/m^3$, $g = 10$, $r = ?$

$$h = \frac{2S\cos\theta}{r\rho g} \Rightarrow r = \frac{2 \times 0.07 \times 1}{0.075 \times 10^3 \times 10}$$

$$r = 0.2\,mm.$$

Q-41 A capillary tube of $r = 0.5\,mm$ and having water ($S = 0.07\,N/m$). Find the difference in Pressure b/w a point which is 5cm below surface of capillary and atmospheric Pressure.

$$\Delta P = P^0 - P^0 - \frac{2S}{R} + \rho g h$$

$\cos\theta = 0$, $r = R$

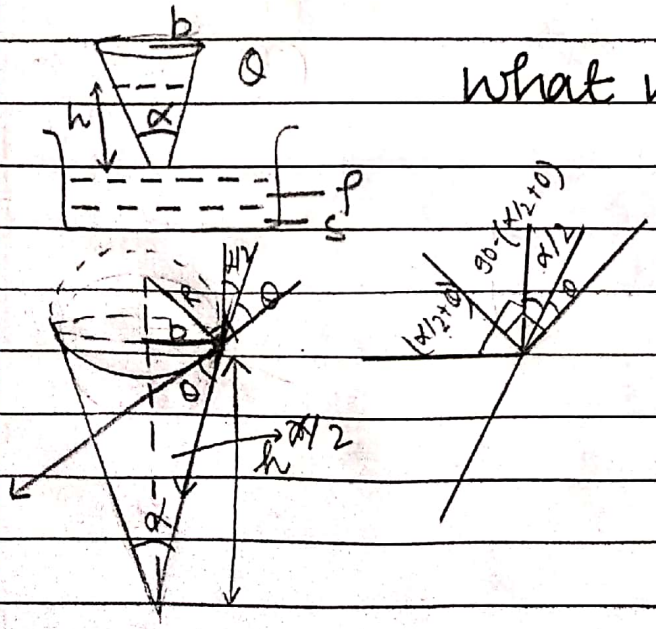
$$\Delta P = \frac{2S}{R} - \rho g h$$

$$= \frac{2 \times 0.075}{0.5 \times 10^{-3}} - 5 \times 10^{-2} \times 10^3 \times 10$$

$$= 300 - 500$$

$$\Delta P = -200\,N/m^2$$

Q-42 JEE 2014 Advance

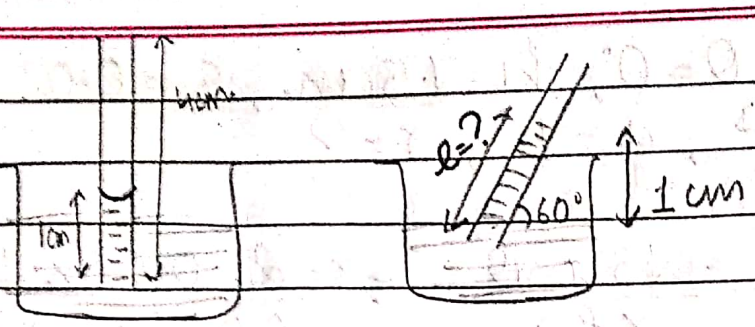


What will be the height?

$$h = \frac{2S\cos(\theta + \alpha/2)}{b\rho g}$$

P.No.-
Dt.-

Q-43



$$\sin 60° = \frac{h}{l} \quad = \quad l = \frac{2}{\sqrt{3}}$$

Q-44 After calculations, $h = 2$ cm, tube is about 4 cm, Cali is tube of insufficient length. if $h = 6$ cm what will happen = ?

Liquid will rise to the top & change its meniscus and contact angle, contact angle will start becoming equal to 90°.